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13. ABSTRACT (Maximum 200 words) On PMD (polarization mode dispersion) effects in long-distance fiber communication systems: (2 papers, 1 conference proceedings article) We have analytically derived the Fokker-Planck equation for the probability distribution of the polarization-mode-dispersion (PMD) vector at any distance in randomly-birefringent fiber communication systems. We then solved this (3+1)- dimensional Fokker-Planck equation with singular initial conditions numerically and obtained the probability distribution of the PMD vector at arbitrary distances once for all. This probability-distribution function is critical for assessing the random-birefringence- induced penalties to long-distance optical communication systems. On vector-soliton collisions and interactions in birefringent fibers: (7 papers, 1 conference proceedings article) We have studied vector-soliton collisions in polarization-maintaining birefringent fibers both analytically and numerically. Mathematically, this collision is governed by the non-integrable coupled nonlinear Schroedinger equations. We have found that vector-soliton collisions in polarization-maintaining fibers have a fractal structure, i.e, this collision depends on the initial velocities very sensitively.			
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## Project Final Report

Re: DEPSCoR Grant USAF F49620-99-1-0174

Project title: Effects of polarization-mode dispersion on fiber  
communication systems

Funding period: 5/1/99 to 4/30/02

Reporting period: 5/1/99 to 4/30/02

Principal Investigator: Jianke Yang, University of Vermont

### I. Summaries of my activities in this period

The past three years have been extremely fruitful, thanks in part for the generous support of this DEPSCoR grant. During this period, I published

22 journal papers (20 published, 2 accepted), and  
2 conference proceedings articles

I have collaborated with 11 colleagues:

David Kaup, University of Central Florida  
Curtis Menyuk, University of Maryland, Baltimore County  
William Kath, Northwestern University  
Boris Malomed, Tel Aviv University, Israel  
Alan Champneys, University of Bristol, UK  
Vladimir Gerdjikov, Institute for Nuclear Research, Bulgaria  
E.V. Doktorov, Steponov Institute of Physics, Belarus  
Ziad Musslimani, University of Colorado, Boulder  
Dmitry Pelinovsky, McMaster University, Canada  
Yu Tan (postdoc)  
X. Chen (postdoc)

I have attended 10 conferences, half of them are optical conferences, and the other half are mathematical conferences. My postdocs have attended optical conferences as well.

I have worked with two postdoc associates who have been a great help to me. We have written 7 papers together (all published). I have had 7 visitors for periods from a few days to two months. Curtis Menyuk and Dave Kaup were two of the visitors.

I have maintained a partnership with University of Maryland at Baltimore County (Curtis Menyuk's research group). Mutual visits have been made each year.

On industry connection, one of my former collaborators (Taras Lakoba) worked in Lucent Technologies. He has visited me recently, and joint work with him on fiber communication problems are under way.

## II. Summary of my research results in this period

1. On PMD (polarization mode dispersion) effects in long-distance fiber communication systems: (2 papers, 1 conference proceedings article)

We have analytically derived the Fokker-Planck equation for the probability distribution of the polarization-mode-dispersion (PMD) vector at any distance in randomly-birefringent fiber communication systems. We then solved this (3+1)-dimensional Fokker-Planck equation with singular initial conditions numerically and obtained the probability distribution of the PMD vector at arbitrary distances once for all. This probability-distribution function is critical for assessing the random-birefringence-induced penalties to long-distance optical communication systems.

2. On vector-soliton collisions and interactions in birefringent fibers: (7 papers, 1 conference proceedings article)

We have studied vector-soliton collisions in polarization-maintaining birefringent fibers both analytically and numerically. Mathematically, this collision is governed by the non-integrable coupled nonlinear Schroedinger equations. We have found that vector-soliton collisions in polarization-maintaining fibers have a fractal structure, i.e., this collision depends on the initial velocities very sensitively.

3. On embedded solitons in physical systems: (5 papers)

We have studied the existence and dynamics of embedded solitons in physical systems both analytically and numerically. We note that 'embedded solitons' is a name we proposed in a 1999 Phys. Rev. Lett. paper for solitary waves which reside INSIDE the continuous spectrum of a wave system. Almost all solitary waves reported before then resided OUTSIDE the continuous spectrum. We have found (through careful analytical calculations and numerical simulations) that embedded solitons under perturbations are semi-stable, i.e., their time evolution depends on the type of initial perturbations they had.

4. On soliton perturbation theory of integrable systems: (4 papers)

The key to a soliton perturbation theory is to find a complete set of eigenfunctions to the linearization operator expanded around the solitons. In this block of work, we have studied four integrable hierarchies: the nonlinear Schroedinger (NLS) hierarchy, the KdV hierarchy, the modified NLS hierarchy, and the derivative NLS hierarchy. We have found that the entire hierarchies of integrable equations share the same set of complete

eigenfunctions. In addition, the linearization operator for any member in a hierarchy can be factorized into the recursion operator (which generates the hierarchy) and the linearization operator of the first member in the hierarchy. We have also derived these complete eigenfunctions explicitly for each hierarchy. These results lay the foundation for a direct soliton perturbation theory for any member in these hierarchies. Indeed, we have applied these results to the integrable fifth-order KdV equation, and analytically studied the dynamics of embedded solitons in perturbed fifth-order KdV equations (see *Stud.Appl.Math.*106, 337, 2001).

#### 5. On miscellaneous subjects related to optical solitons: (4 papers)

Here, we have studied various problems related to spatial and temporal optical solitons. For instance, we have studied the internal oscillations and instability characteristics of  $(2+1)$  dimensional solitons in a saturable nonlinear medium. Internal modes and unstable modes are studied in detail. Contrary to conventional wisdom, we found that high-power solitons are actually less unstable. We have also studied the transverse instability of strongly coupled dark-bright Manakov vector solitons. The entire instability curve was determined. It was found that dark-soliton transverse instability is significantly reduced in Kerr media by strong coupling to a bright soliton. We have also studied internal oscillations and radiation damping of vector solitons in birefringent fibers. Our analytical formulas for internal modes and their radiation-damping rates are in excellent agreement with the numerical results. Lastly, we have examined discrete solitons in the coupled Ablowitz-Ladik model, and determined the regions where the coupled solitons are stable.

### III. Contributions of these results to fiber communications and applied mathematics

1. Our determination of the probability distribution of PMD vectors at arbitrary distances involved a non-trivial derivation and asymptotic analysis of the Fokker-Planck equation. In addition, we employed an ingenious split-step numerical method to solve this  $(3+1)$ -dimensional Fokker-Planck equation with singular initial conditions. Thus, our results contributed to the theories of both the stochastic differential equations and numerical methods. More importantly, from the engineering point of view, our results have filled a gap on the PMD vector's probability distribution in the literature. Our results are critical for the assessment of PMD-induced penalties on fiber communication systems. They have generated considerable interest in Lucent Technologies, whose engineers have contacted us and asked us to explain the technical details of our papers to them.

2. Our results on fractal structures of vector-soliton collisions in birefringent fibers have attracted the attention of many physicists and mathematicians. Mathematically, the existence of such fractal structures is a surprising result in the theory of non-integrable nonlinear wave equations (soliton collisions in integrable systems are elastic, thus this kind of fractal structures could never exist in integrable systems). Physically, this fractal

dependence of the collision could have important applications and implications. Physicists at Princeton University have expressed their interests in doing experiments on vector-soliton collisions in birefringent fibers to confirm our theoretical results.

3. The discovery of embedded solitons is significant due to their novel property that they reside INSIDE the continuous spectrum of the wave system (almost all solitary waves reported before reside outside the continuous spectrum). Embedded solitons are a new class of solitary waves which start to receive more and more attention. Our results on embedded solitons have shed much light on the existence and dynamics of embedded solitons in various physical systems such as generalized second-harmonic-generation system and generalized KdV system. We have established both theoretically and numerically that embedded solitons are semi-stable, i.e., their evolution depends on the type of initial perturbation imposed on them. This interesting property may be ideal for certain applications such as switching.

4. The finding that linearization operators of an entire integrable hierarchy share the same complete set of eigenfunctions and can be factorized into the recursion operator and the linearization operator of the first member in the hierarchy is an elegant mathematical result on integrable equations. It reveals new hidden structures of integrable systems. From a physical point of view, many physical problems are governed by perturbed integrable equations such as the perturbed NLS equation and perturbed derivative NLS equation. The complete sets of eigenfunctions for the NLS, KdV, modified KdV and derivative NLS hierarchies we have derived lay the mathematical foundation for soliton perturbation theories for any member in those hierarchies. They will find applications to a whole range of problems such as ultra-short pulse propagation in optical fibers (where a perturbed derivative NLS equation is relevant).

#### IV. List of published and accepted papers in this period

##### 1. On PMD effects in fiber communication systems

- (1) Y. Tan, J. Yang, W.L. Kath and C.R. Menyuk, "Transient evolution of the polarization dispersion vector's probability distribution." J. Opt. Soc. Am. B. 19, 992 (2002).
- (2) J. Yang, W.L. Kath and C.R. Menyuk, "Polarization mode dispersion probability distribution for arbitrary distances." Opt. Lett. 26, 1472 (2001).
- (3) J. Yang, W.L. Kath, and C.R. Menyuk, "PMD probability distribution for arbitrary distances." Proceedings of CLEO, pp58-59 (2000).

##### 2. On vector-soliton collisions and interactions in birefringent fibers

- (1) J. Yang and Y. Tan, "Fractal structure in the collision of vector solitons." *Phys. Rev. Lett.* 85, 3624 (2000).
- (2) J. Yang and Y. Tan, "Fractal dependence of vector-soliton collisions in birefringent fibers." *Phys. Lett. A.* 280, 129 (2001).
- (3) Y. Tan and J. Yang, "Complexity and regularity of vector-soliton collisions." *Phys. Rev. E.* 64, 056616 (2001).
- (4) Y. Tan and J. Yang, "Resonance and phase induced window sequences in vector soliton collisions." *Phys. Lett. A.* 288, 309 (2001).
- (5) J. Yang, "Interactions of vector solitons." *Phys. Rev. E* 64, 026607 (2001).
- (6) J. Yang, "Suppression of Manakov-Soliton Interference in Optical Fibers." *Phys. Rev. E.* 65, 036606 (2002).
- (7) V. S. Gerdjikov, E. V. Doktorov and J. Yang, "Adiabatic Interaction of N-Ultrashort Solitons: Universality of the Complex Toda Chain Model." *Phys. Rev. E.* 64, 056617 (2001).

### 3. On embedded solitons in physical systems:

- (1) D.E. Pelinovsky and J. Yang, "A normal form for nonlinear resonance of embedded solitons." *Proc. Roy. Soc. Lond. A.* 458, 1469-1497 (2002).
- (2) Y. Tan, J. Yang and D.E. Pelinovsky, "Semi-stability of embedded solitons in the general fifth-order KdV equation." *Wave Motion* 36, 241 (2002).
- (3) J. Yang, B.A. Malomed, D.J. Kaup and A.R. Champneys, "Embedded solitons: a new type of solitary waves". *Mathematics and Computers in Simulation*, 56, 585 (2001).
- (4) A.R. Champneys, B.A. Malomed, J. Yang and D.J. Kaup, "Embedded solitons: solitary waves in resonance with the linear spectrum". *Physica D* 152, 340 (2001).
- (5) Yang, J., "Dynamics of embedded solitons in the extended KdV equations." *Stud. Appl. Math.* 106, 337 (2001).

### 4. On soliton perturbation theory of integrable systems:

- (1) X. Chen and J. Yang, "A direct perturbation theory for solitons of the derivative nonlinear Schroedinger equation and the modified nonlinear Schroedinger equation." *Phys. Rev. E.* 65, 066608 (2002) [also, *Virtual J. Ultrafast Sci.* Volume 1, issue 2 (2002)].

(2) J. Yang, "Eigenfunctions of linearized integrable equations expanded around an arbitrary solution." Stud. Appl. Math. 108, 145 (2002).

(3) J. Yang, "Structure of linearization operators of the Korteweg-de Vries hierarchy equations expanded around single-soliton solutions." Phys. Lett. A 279, 341 (2001).

(4) J. Yang, "Complete eigenfunctions of linearized integrable equations expanded around a soliton solution". J. Math. Phys. 41, 6614 (2000).

5. On miscellaneous subjects related to optical solitons:

(1) J. Yang, "Internal oscillations and instability characteristics of (2+1) dimensional solitons in a saturable nonlinear medium." To appear in Phys. Rev. E (August, 2002).

(2) Z.H. Musslimani and J. Yang, "Transverse instability of strongly coupled dark-bright Manakov vector solitons." Opt. Lett. 26, 1981 (2001).

(3) Pelinovsky, D.E. and Yang, J., "Internal Oscillations and radiation damping of Vector Solitons." Stud. Appl. Math. 105, 245 (2000).

(4) B.A. Malomed and J. Yang, "Solitons in coupled Ablowitz-Ladik chains." To appear in Phys. Lett. A.